# Ph.D. Dissertation Defense Announcement
# Mechanical and Aerospace Engineering Department
# University of Texas at Arlington

## STRESS ANALYSIS OF FUNCTIONALLY GRADED MATERIALS


By

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## Abstract


A functionally graded material (FGM) is an inhomogeneous material whose material properties vary continuously. Since FGM's material properties are continuous at the interface of the two materials, unlike conventional composites whose material properties are discontinuous at the interface, it is best suited to prevent catastrophic failure, stress concentration, and residual stresses.

Literature reviews revealed that most of the research on FGMs was a 2-D analysis, especially on FGM plates and beams. Our research focuses on 3-D stress analysis for an elastic medium that contains a spherical inclusion made of an FGM embedded in a matrix phase. To our best of knowledge, this is the first attempt to analyze inclusion-type FGMs semi-analytically. The material properties are assumed to vary linearly, which is suitable for a solid or hollow sphere, although this assumption can be relaxed to the exponential or power-law variations. We have considered three different scenarios, which are (1) 3-D thermal stress analysis of an infinitely extended medium which contains a spherical FGM inclusion with a constant heat source in the FGM inclusion, (2) 3-D thermal stress analysis of an infinitely extended medium which contains a spherical FGM inclusion subject to constant heat flux at the far fields, and (3) 3-D elastic stress analysis of an infinitely extended medium which contains a spherical FGM inclusion subject to constant strain at the far fields. Each one of them is discussed separately.

The 3-D temperature distribution is sought in the heat source problem due to the constant heat source in the inclusion, and the 3-D heat conduction equation was solved analytically. The material properties of Ni-ceramic are taken as an example, and we plotted the graph of temperature distribution. It is found that the graph is smooth at the interface of FGM-matrix, unlike the two-phase conventional composites, where there is an abrupt change in the slope at the interface. Furthermore, this temperature distribution is substituted into the elasticity equation to solve Navier's equation of displacement. The stress equilibrium equation is converted into a second-order ordinary differential equation using tensor analysis. A computer algebra tool, Mathematica, was used to solve the differential equation to obtain the exact solution. However, the output was too lengthy and not practically useful. Therefore, we have adopted the least square method and plotted the results. The least square method is used to solve the differential equation because of its simplicity. The results are then compared with previously published articles, where they give an excellent comparison. It has also been shown that the von Mises stress is continuous at the interface of FGM-matrix, unlike the two-phase conventional composites where there is a considerable gap at the interface.

In the heat flux problem, the same approach has been used as in the heat source problem we have considered. The only difference is that the temperature distribution is not due to the heat source but due to the constant heat flux at the far fields. We obtained the exact solution of the 3-D heat conduction equation, and the result of temperature distribution is plotted using the material properties of glass-ceramic. The temperature distribution is substituted in the stress equilibrium equation, and Mathematica is employed to solve the differential equation analytically. However, Mathematica could not solve the set of two simultaneous differential equations. Again, we have adopted the least square method and plotted the results. The results are then compared with the two-phase conventional composite.
In the constant strain problem, the same approach is used as in the heat flux problem we have considered, but instead of considering constant heat flux at the far fields, constant strain at the far fields is considered. Here, the total strain is decomposed into the deviatoric and hydrostatic strains. Both of them are analyzed separately. In the 3-D stress analysis due to hydrostatic strain, we obtained the exact solution for the stress equilibrium equation, plotted the results, and compared it with the two-phase conventional composites. 3-D stress analysis due to the deviatoric strain is also performed by solving the stress equilibrium equation using the least square method. The results are shown and compared with the two-phase conventional composites.

The solution obtained in this research can be directly used to assess the performance of composites where multiple FGM-made inclusions are distributed in a matrix phase.

Finite element tools such as Ansys or Abaqus are not suitable for our research because they cannot handle the continuous variation of the material properties. Also, they cannot handle an infinitely extended matrix