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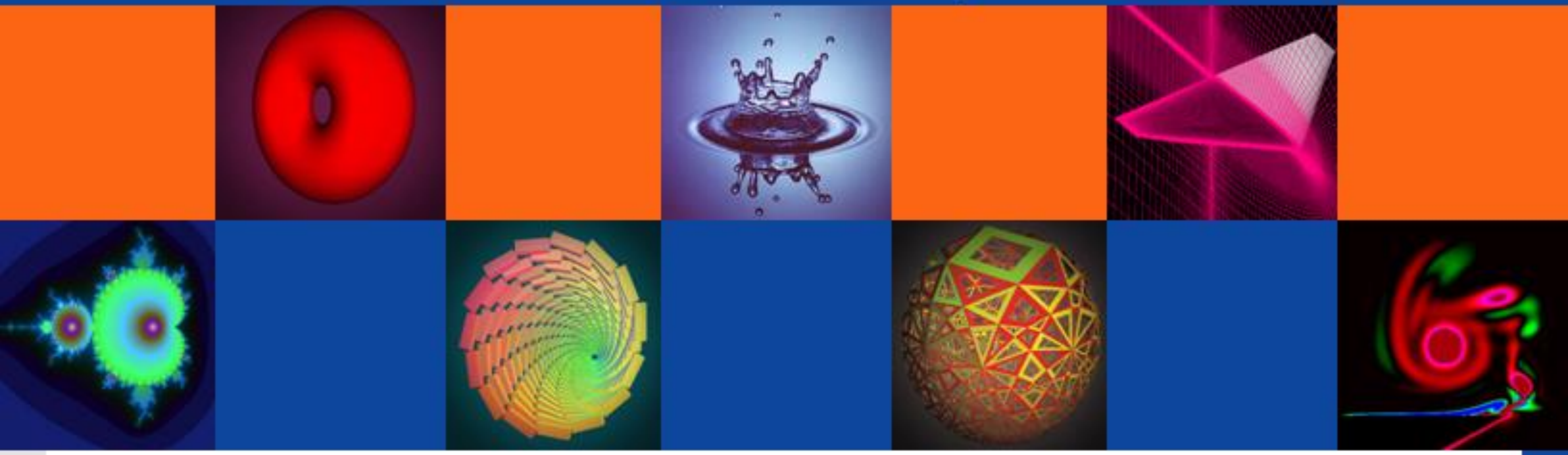

DNS Study on Mechanism of Chaos in Late Flow Transition

Yong Yang

Yonghua Yan

Chaoqun Liu

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DNS Study on Mechanism of Chaos in Late Flow Transition

Yong Yang, Yonghua Yan, Chaoqun Liu
University of Texas at Arlington

Abstract

The mechanism of chaos in late boundary layer transition is a key issue of the laminar-turbulent transition process. A careful study on the characteristic of chaos is carried out by high order direct numerical simulation (DNS). The process of flow chaos was originally considered as a result of large background noise and non-periodic spanwise boundary conditions. It was addressed that the large ring structures are affected by background noises first, and then the change of large ring structures affect the small scale vortices quickly, which directly lead to chaos and formation of turbulent flow. However, according to our DNS observation, the loss of symmetry starts from the middle level vortex rings while the top and bottom rings are still symmetric. The non-symmetric structure of second level vortex rings will influence the small scale vortices at the boundary layer bottom quickly. The loss of symmetry at the bottom of the boundary layer quickly spreads to upper level through ejections. This will lead to chaos of the whole flow field. Therefore, the internal instability of multiple level vortex ring structures, especially the middle ring cycles, is the main reason for the process of flow chaos, but not the large background noise. A new numerical simulation and theoretical analysis is carried out on the multiple level vortex ring package stability. The top package is found stable since it is laid out near the inviscid area and the bottom package is found stable since it is constrained by the solid surface. The middle vortex ring package is found most unstable since there is no constrains to the package. The current analysis is focused on the stability of two rotation cores overlapping, which are moving closer and closer. It is found that the flow becomes more unstable when the two cores are moving closer and closer. More details of the stability analysis will be reported in the full paper.

Keywords: DNS, Turbulence, Transition, Chaos, Boundary layer

1. Preliminary Results and Analysis

1.1 Velocity distribution

The distribution of averaged streamwise velocity are given in Fig 1 along the normal grid lines at the center plane of a ring-like vortex, whose streamwise position is at $x = 491.1\,\delta_{in}$.

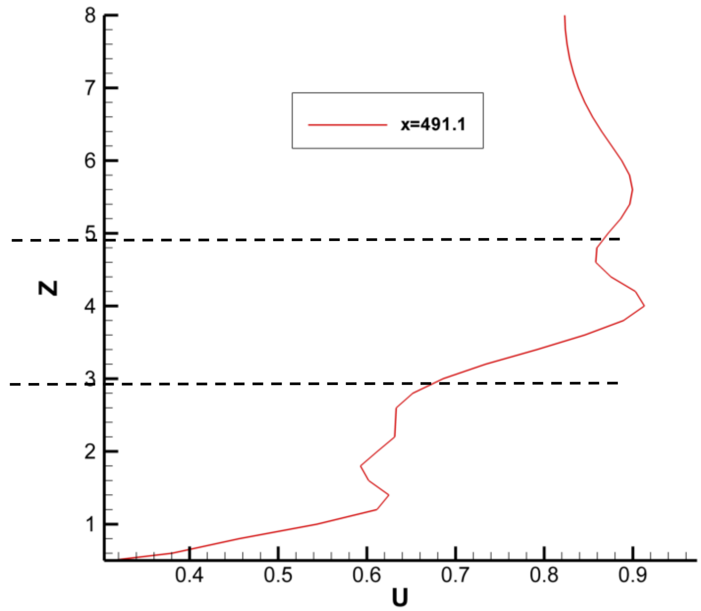
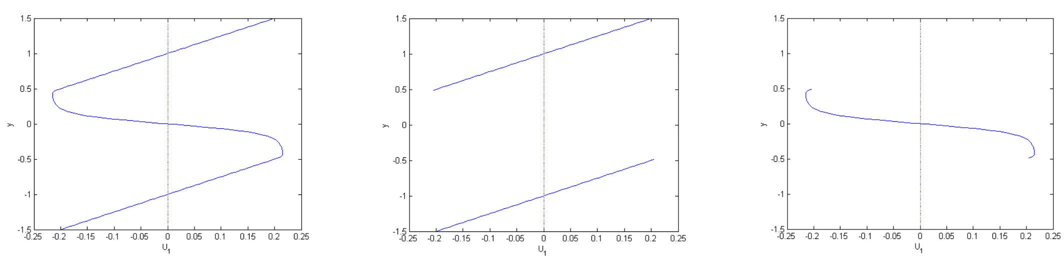


Figure 1 Streamwise Velocity Profile

The approximations of the base velocity profiles are given in three cases, see Figs 2-4. The distance between two rotation centers are increased from Case 1 to Case 3.

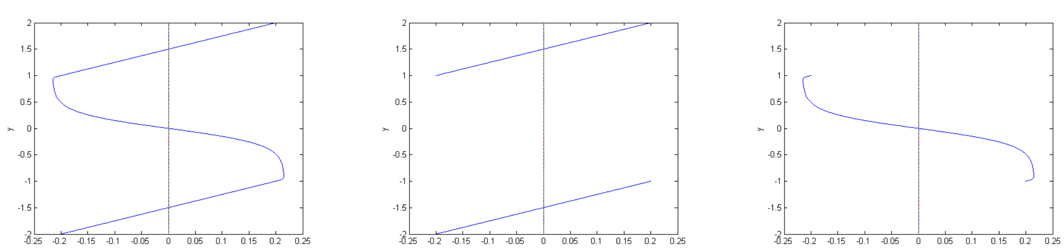


(a) Velocity distribution

(b) Rotational part

(c) Shear part

Figure 2: Case 1

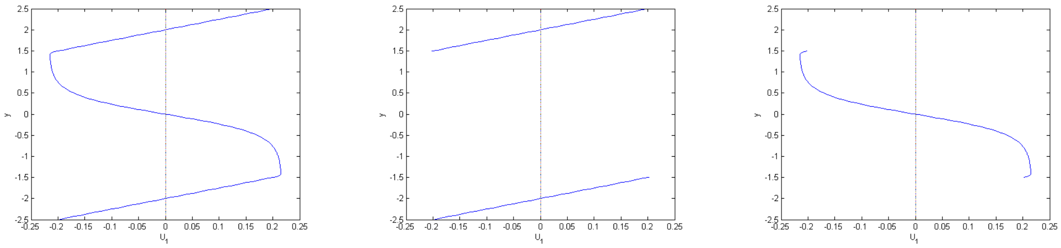


(a) Velocity distribution

(b) Rotational part

(c) Shear part

Figure 3: Case 2



(a) Velocity distribution **(b) Rotational part** **(c) Shear part**

Figure 4: Case 3

1.2 Derivation of Linear Stability Equation

$$\begin{cases} \dfrac{\partial V}{\partial t} + V \cdot \nabla V = -\nabla p + \dfrac{1}{Re}\nabla^2 V \\ \nabla \cdot V = 0 \end{cases} \tag{1}$$

Equation (1) denotes the incompressible and non-dimensional Navier-Stokes equations in which, $V = (u, v, w)$ is the velocity vector. Considering that

$$q(x, y, t) = q_0(y) + q'(x, y, t) \tag{2}$$

where q can be specified as (u, v, w, p), and $q_0 = (u_0, v_0, w_0, p_0)$ which represents the value of mean flow, and q' denotes the corresponding linear perturbation. By eliminating the second order perturbation terms, the linearized governing equation for small perturbations can be written as,

$$\begin{cases} \dfrac{\partial V'}{\partial t} + (V_0 \cdot \nabla)V' + (V' \cdot \nabla)V_0 + \nabla p' = \dfrac{\nabla^2 V'}{Re} \\ \nabla \cdot V' = 0 \end{cases} \tag{3}$$

As a first step, a localized 2-D incompressible temporal stability for shear layer is studied. Actually, it relates to the distance among two neighboring vortices in the central streamwise plane. Assume the normal mode is

$$V' = \hat{V}(y)e^{i(\alpha x+\beta z-\omega t)} + c.c. = \hat{V}(y)e^{i\alpha(x+\frac{\beta}{\alpha}z-ct)}$$

$$p' = \hat{p}(y)e^{i(\alpha x+\beta z-\omega t)} + c.c. = \hat{p}(y)e^{i\alpha(x+\frac{\beta}{\alpha}z-ct)} \quad (4)$$

$$c = \frac{\omega}{\alpha}$$

where the parameter α is given, which is real and set according to the averaged distance between the new generated rings, and c should be a complex number. Plugging Equation (4) in Equation (3) yields

$$\begin{aligned} L\hat{u} &= \mathrm{Re}(Du_0)\hat{v} + i\alpha \mathrm{Re}\hat{p} \\ L\hat{v} &= \mathrm{Re}(D\hat{p}) \\ L\hat{w} &= i\beta \mathrm{Re}\,\hat{p} \\ i(\alpha\hat{u} + \beta\hat{w}) + D\hat{v} &= 0 \end{aligned} \quad (5)$$

where $L = \{D^2 - (\alpha^2 + \beta^2) - i\,\mathrm{Re}(\alpha u_0 - \omega)\}$, and $D = \frac{d}{dy}$

Considering in 2D case (without w), and by eliminating $\hat{u}, \hat{p}$, we can obtain the standard O-S equation on $\hat{v}$,

$$(D^2 - \alpha^2)^2\hat{v} - i\alpha \mathrm{Re}[(U_0 - c)(D^2 - \alpha^2) - D^2U_0]\hat{v} = 0 \quad (6)$$

Equation (6) is about $\hat{v}$, but we need to get the value of c. The value of c determines the property of stability of the equation. Let $c = c_r + ic_i$, if $c_i > 0$, then the disturbance will continuously grow and the flow would be instable. While if c_r is greater, the disturbance will grow faster and the flow would be more unstable. But if $c_i < 0$, the flow would be stable.

1.3 Chebyshev Spectral Method for Linear Stability Analysis

Spectral methods have a significant impact on the accurate discretization of both initial value problems and eigenvalue problems. And spectral method with Chebyshev polynomials has been advantageous, especially in stability analysis of fluid mechanics.

In this stability analysis, the function $\hat{v}$ could be approximated by Chebyshev expansion,

$$\hat{v}(y) = \sum_{n=0}^{\infty} a_n T_n(y) \approx \sum_{n=0}^{N} a_n T_n(y) \quad (7)$$

where N is the number of Chebyshev polynomials used to approximate the velocity profile, T_n are the Chebyshev polynomials and a_n are the coefficients.

After some algebraic work, Equation (6) yields

$$(-U\alpha^2 - U'' - \frac{\alpha^3}{i\mathrm{Re}})\hat{v} + (U + \frac{2\alpha}{i\mathrm{Re}})\hat{v}'' - \frac{1}{i\alpha\mathrm{Re}}\hat{v}'''' = c(\hat{v}'' - \alpha^2\hat{v}) \tag{8}$$

By approximating $\hat{v}$ with a certain Chebyshev expansion, Equation (8) gives

$$\sum_{n=0}^{N}[(-U\alpha^2 - U'' - \frac{\alpha^3}{i\mathrm{Re}})T_n + (U + \frac{2\alpha}{i\mathrm{Re}})T_n'' - \frac{1}{i\alpha\mathrm{Re}}T_n'''']a_n = c\sum_{n=0}^{N}a_n(T_n'' - \alpha^2 T_n) \tag{9}$$

If there is no disturbance at the boundary and it will be free stream outside the domain (a, b), then we have the corresponding boundary condition for function $\hat{v}$ as $\hat{v}(a) = \hat{v}(b) = 0$ and $D\hat{v}(a) = D\hat{v}(b) = 0$.

Applying Equation (9) on the whole grids with boundary conditions above, a matrix form of generalized eigenvalue problem is given by

$$Aa^{(\hat{v})} = cBa^{(\hat{v})} \tag{10}$$

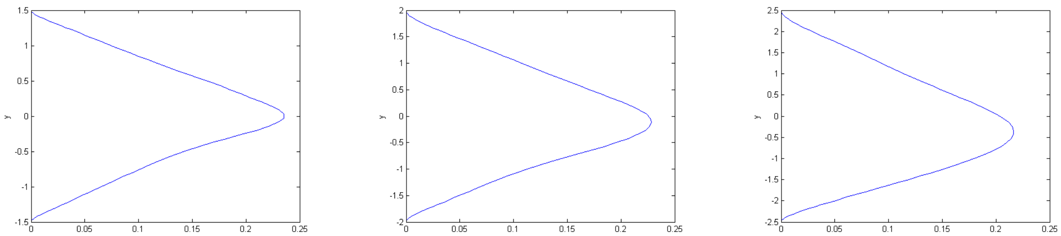
where both A and B are the coefficients' matrix and the vector $a^{(\hat{v})}$ denotes the vector of $\{a_n\}$. c becomes unknown in the generalized eigenvalue of Equation (10).

1.4 Stability Analysis to the Three Velocity Profiles

By solving the general eigenvalue problem for the standard Orr-Sommerfeld equation -- Equation (9) and (10), at $\mathrm{Re} = 1000$ which follows the configuration in the DNS case, the physical solution of the eigenvalue c is obtained. It shows that these three cases are all unstable. Tab.1 gives the value of generalized eigenvalue c in three cases and Figs 2-4 give the corresponding profile of eigenvector functions.

case	Distance between two rotation centers	Imaginary part of c
1	2.0	0.71482
2	3.0	0.26741
3	4.0	0.20694

Table 1 Results of c_i for the velocity profile in three cases at Re=1000, $\alpha = 1.0$



(a) Case 1 (b) Case 2 (c) Case 3

Figure 5: Corresponding profile of the eigenvector function for three cases

By comparison, we can find the image part of c is the greatest in Case 1 and is the least in Case 3. That means the disturbance will grow faster in Case 1 and slower in Case 3. Note that the distance between two rotation centers is growing from Case 1 to Case 3 , and it is reasonable that the disturbance will grow faster and the flow would be more unstable if two rotation centers are closer to each other.

2. Some conclusions and future work

The mechanism of chaos in late boundary layer transition is a key issue of the laminar-turbulent transition process. The internal instability of multiple level vortex ring structures, especially the middle ring cycles, is the main reason to cause the asymmetry and then flow chaos, but not the large background noise according to the observation of our DNS computation. A new numerical simulation and theoretical analysis is carried out on the multiple level vortex ring package stability. A two level rotation core overlapping is studied and it is found that the flow becomes more unstable when the two cores are moving closer and closer. More details of the stability analysis will be reported in the full paper.

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