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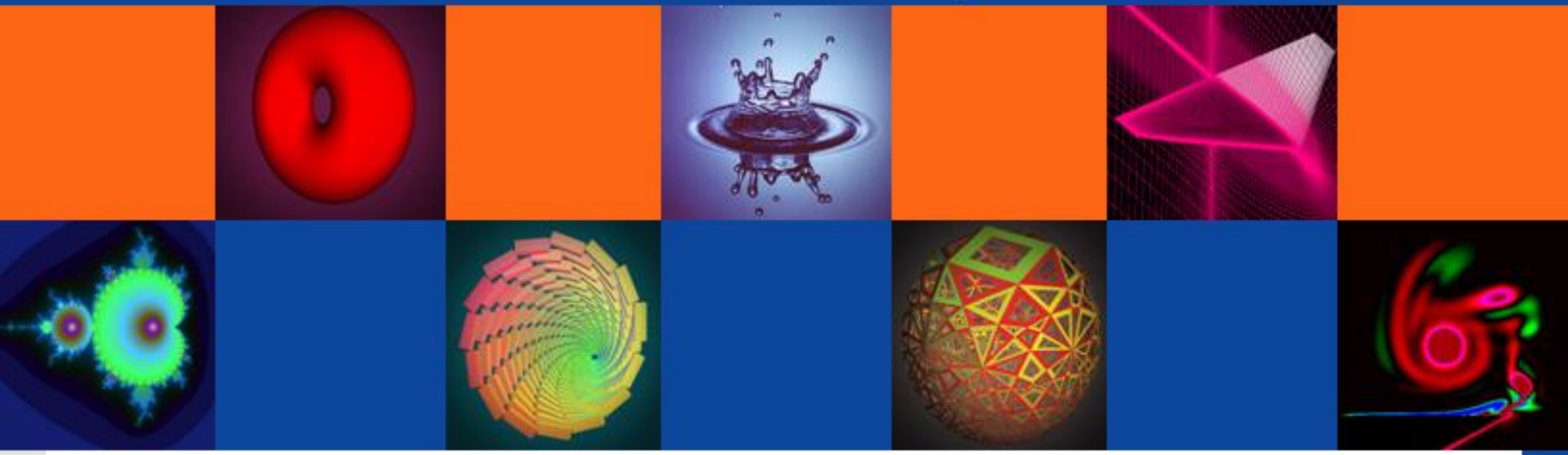

DNS Study on Small Length Scale in Turbulent Flow

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DNS Study on Small Length Scale in Turbulent Flow

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Abstract

According to our observation of turbulence generation, all small length scales are generated by shear layer instability. The size of small vortices should be determined by the smallest stable shear layer, e.g. smallest shear layer like laminar sub-layer in the wall bounded flow. This paper provides a new estimation of the smallest length scale by using the shear layer stability analysis, which is different from Kolmogorov micro scales, or $0(\mathrm{Re}^{-3/4})$

I. Introduction

Classical turbulence theory about vortex chains was given by Richardson. He has a famous poem that "Big whirls have little whirls, which feed on their velocity; And little whirls have lesser whirls, and so on to viscosity in the molecular sense." However, the vortex chain generated by large vortex breakdown is never observed. As shown by our DNS, turbulence has different size of vortices from the large to small. However, they are all generated by shear layer instability (K-H type) without exception and no vortex breakdown is observed. In fact, no one ever observed the eddy cascade by instrument or computation.

The Richardson energy cascade and vortex breakdown was accepted by Kolmogorov and "vortex breakdown' as the foundation of Kolmogorov's hypotheses. The famous Kolmogorof scale is given by Russian Mathematician Kolmogorof in 1941. The scale is obtained by dimensional analysis. Assume the velocity and length related to the largest eddy are $L\ and\ U$, ν is the viscosity, ε is the kinetic energy, the velocity and length related to the smallest eddy are $V\ and\ \eta$, we will have the energy relation:

$\varepsilon \sim \frac{U^2}{L/U} \sim \frac{U^3}{L} \sim \nu S_{ij} S_{ij} \sim \nu(\frac{V^2}{\eta^2})$ which means all energy, transported from largest eddy to the smallest eddy, is dissipated. Note that $\mathrm{Re}_\eta = \frac{V\eta}{\nu} \sim 1$,

$$\frac{U^3}{L} \sim \nu \frac{V^2}{\eta^2} \rightarrow \frac{U^3 L^3}{\nu^3} \sim \frac{V^2 \eta^2}{\nu^2} \cdot \frac{L^4}{\eta^4} \rightarrow \mathrm{Re}^3 \sim (\frac{L}{\eta})^4$$

$$\because \mathrm{Re} = \frac{UL}{\nu}\ and\ \mathrm{Re}_\eta = \frac{V\eta}{\nu} \sim 1 \qquad (1)$$

Finally,

$$(\frac{\eta}{L})^4 = \mathrm{Re}^3\ or\ \frac{\eta}{L} = \mathrm{Re}^{-3/4}$$

$\frac{\eta}{L}$ represents the ratio of the Kolmogorov micro scale over the macro scale, which is very small, e.g. if $\mathrm{Re}=10^8$, the ratio could be $\frac{\eta}{L}=10^{-6}$ which cannot be resolved by any modern computers for a 3-D DNS.

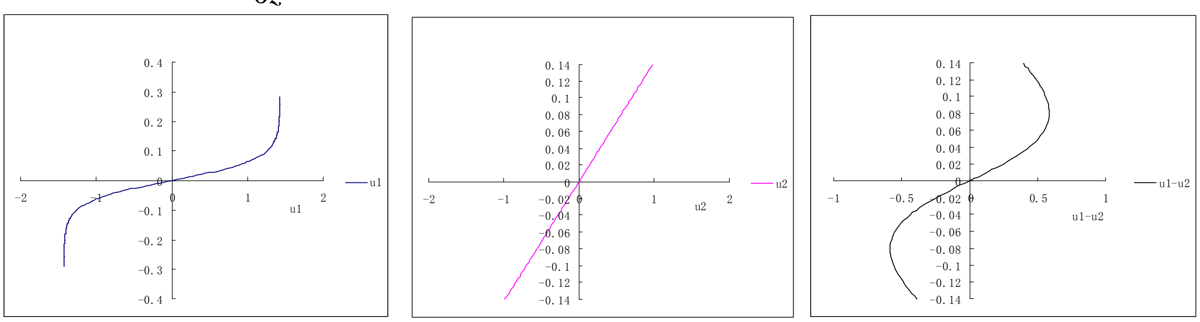
Kolmogorov's hypothesis is based on dimensional analysis and seems to have no possibility to be incorrect. However, there are still some questions:

1) If the Kolmogorov micro scale is generated by "large vortex breakdown", it needs 20 cycles to break down for $\mathrm{Re}=10^6$. However, no one ever observed such 20 generations of vortex breakdown cycles, even a single one.
2) When the Reynolds number becomes very large the Kolmogorov's small scale becomes extremely small. However, no one observed the real Kolmogorov micro scales either by experiment or DNS.

We believe the Kolmogorov's hypothesis and his micro-scale need to be revisited.

II Shear Layer Instability and Turbulence Small Scales

According to our DNS observation, all small vortices are generated by shear layer instability, the smallest scale should be the smallest stable shear layer. We have done some 1-D and axial-symmetric shear layer stability analysis and found that even we have the inflection points, the shear layer still could be stable if the shear strength or $\frac{\partial u}{\partial z}$ is not large enough.



(a) Shear layer with inflection point (u1) **(b) Rotational part (u2)** **(c) Shear part (u1-u2)**

Figure 1: Shear layer velocity decomposition

We pick a typical 1-D shear layer described by following functions: $\vec{U}_1 = a\ \tanh(by)$

As we know, the existence of inflation point makes the velocity profile to be unstable for inviscid fluid flow. However, Table 1. gives the results of c_i (positive is unstable and negative is stable) for the satiability analysis for standard O-S equation at different values of b for the velocity profile *U=a tanh(by)* at *Re=100* and *a=1.43*. It shows that although there is an inflation point, where the second derivative equals 0, for every curve, with relative small value for b the flow could be stable. Thus, the conclusion can be made that the stability is not only related to the inflation point of the velocity profile but also related to the slope of the curve (the curve is steeper with larger b) for viscous flow. To make a flow be unstable, the existence of inflation point is just a necessary condition, and the velocity profile should be steep enough. The numerical study is carried out in an axis-symmetric coordinate.

b	0.1	0.2	0.6	0.9	1.0	2.0	3.0	4.0
c_i	<0	<0	<0	<0	<0	0.1186	0.3965	0.5517

Table 1. Results of c_i at different values of b for the velocity profile U=1.43 tanh(b*y) at Re=100, α=1.0

According to our observation, the size of the smallest stable shear layer should be considered as the smallest scale.

For any wall bounded turbulent flow, there is always a laminar sub-layer which is the smallest stable shear layer:

$$z^+ = 1 - 10,\ z^+ = \frac{z}{\delta_v}, \delta_v = \frac{\nu}{u_\tau}, u_\tau = \sqrt{\frac{\tau_W}{\rho}} \tag{2}$$

On the other hand, Kolmogorov's smallest scale was derived by dimensional analysis but has not been confirmed by any experiment or computation yet when Reynolds number is large, which is the assumption given by Kolmogorov. The serious weakness of classical theory given by Richardson and Kolmogorof is that nobody ever observed. As a roughly estimate, we need 20 vortex breakdowns to get Kolmogorov scale, but we even cannot see a single one. As the experiment tools are so powerful and the visualization technology is so advanced nowadays, it is very hard to believe we still cannot detect the vortex breakdown process. The only conclusion we can believe is that the classical theory on turbulence generation may be incorrect and Kolmogorov's smallest scale may not exist.

III A Counterexample to Kolmogorov Scale

3.1 Tip vortex relaminarization – a contradiction to classical turbulence theory

As reported by both experiment and large eddy simulation (Cai te al, 2008; Figures 2 and 3), the tip vortex was developed on a 3-D airfoil and becomes turbulent on the airfoil surface. According to the classical trubulence theory, the tip vortex should be more turbulent and has smaller length scales as leaving the airfoil surface and traveling to the free stream since the Reynolds number becomes larger. As Reynolds number becomes larger, the length scale will be smaller according to Kolmogorov's theory and many smaller vortices will be found by "vortex breakdown" according to Richardson's vortex cascade. Unfortunately, the observation by both experiment and LES shows the oposite. The flow is relaminarized and all small length scales disappear. This example strongly oposes classical turbulence theory and strongly supports our new theroy that is "turbulence is not generated by "vortex breakdown" but by "shear layer instability" since strong shear layers disapper when the tip vortex left the airfoil surface. This is apparently an counterexample to Kolmogorov's miscroscale since we find the smallest scale becomes 1 as the flow re-laminarized. According to Kolmogorov's miscroscale hypothesis, $\frac{\eta}{L} = \mathrm{Re}^{-\frac{3}{4}}$, the Reynolds number should be 1, which is impossible according to the definition of Reynolds number.

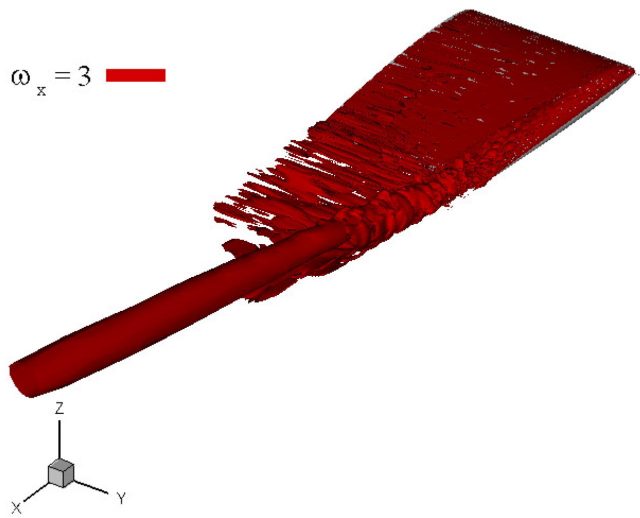


Figure 2: Instantaneous field of axial vorticity: Iso-surface of vorticity component ω_x=3

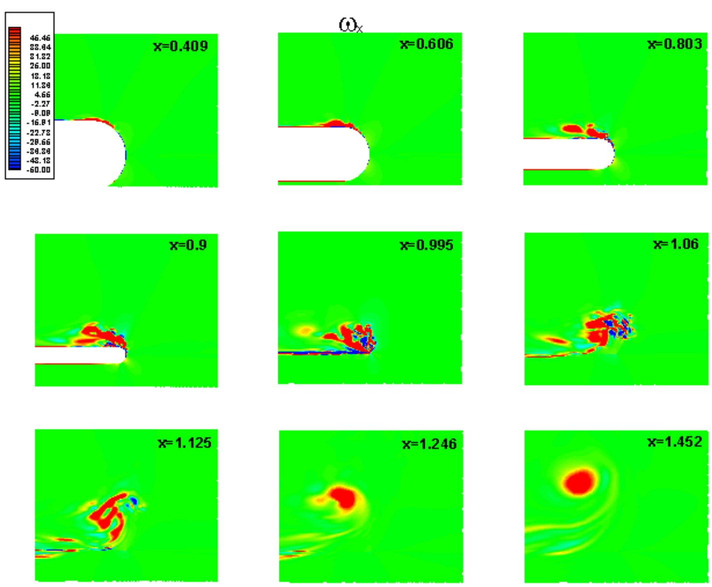


Figure 3. Contours of streamwise vorticity ω_x in cross planes at different locations (Tip vortex left the wall surface at x=1.00)

3.2 Important conclusions

Although Kolmogorov's micro scale is derived based on dimensional analysis, there is no evidence by experiment or DNS to confirm Kolmogorov's vortex breakdown and micro scale (for smallest vortex, Re=1.) According to our observation, all small scales are generated by shear layer instability, the smallest length scale should be the smallest stable shear layer.

It is hard to prove a theory correct, but a counterexample is good enough to overthrow a theory. The tip-vortex is a counterexample to Kolmogorov's micro scale hypothesis.

3.3 Future work

More detailed description of the DNS results and more deep analysis will be given in the final AIAA paper.

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