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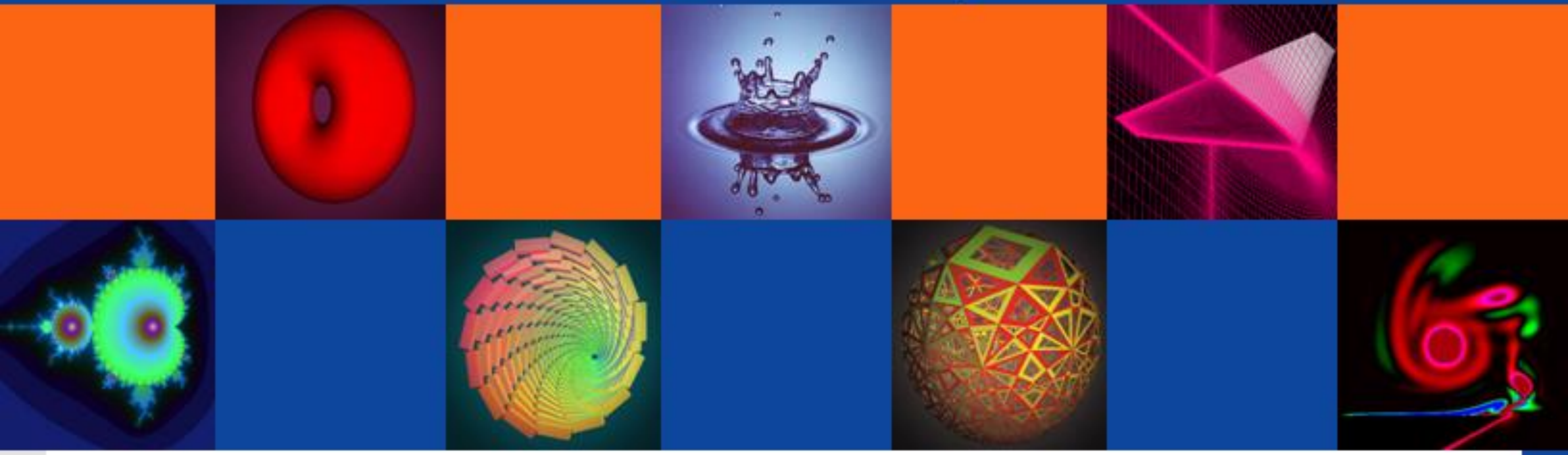

# DNS Study on Role of Linearly Unstable Modes in Flow Transition

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# DNS Study on Role of Linearly Unstable Modes in Flow Transition

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## Abstract

**Flow transition from laminar to turbulent flow is widely considered as caused by linear unstable modes through absolute or convective instability. However, our DNS study shows that is not the case. In our previous AIAA paper, the nature of flow transition is described as an inherent property of fluid flow that fluid cannot tolerate shear layer and shear must transfer to rotation when the Reynolds number is large, which will lead to flow transition. In the current DNS study, three inflow disturbances, T-S waves, blowing jets and random noises, are tested separately and carefully compared. The development of disturbances and late flow transition are investigated by DNS. It is found that the late flow transition are all caused by shear layer instability including the vortex ring formation, multilevel shear layers, multiple level sweeps and ejections. The role of all unstable modes are same which is to trigger the vorticity rollup and change the base flow profile to have inflection points. Since all disturbances, like dust, gust, noise, mosquito, fly, sands, etc., can trigger the vorticity rollup, the idea to use control of linear modes to control flow transition is hard to get success and the key issue is to avoid the vorticity rollup, like use of suctions.**

## I. Introduction

It is widely accepted that flow transition has four stages which are receptivity, linear instability, non-linear instability, vortex breakdown to turbulence. The key issue was thought the development of linear unstable modes. Flow transition is thought as caused by these linearly unstable modes through absolute or convective instability. Then removal or reduction of the linear unstable modes becomes the key technology to control the flow transition. However, according to our DNS study on flow transition, this is not the case. Our DNS study found that

1) Fluid motion can be decomposed as a shear part and rotation part ignoring the translation part. The shear part is unstable when the Reynolds number is large enough and the rotational part is linearly stable. Therefore, flow has trend to change the shear part to rotation when away from the wall. The laminar flow is dominated by shear part and the turbulent flow is dominated by rotation. In the interior flow filed, once the shear layer is formed, flow transition is not avoidable when the Reynolds number is large enough as the shear is continuously stretched. The rotation we defined here is a rotation core like rigid body which is linearly stable.
2) The vorticity is large near the wall surface where the shear is dominant. Although the linearly unstable modes are important for flow transition, they are small, cannot form vortex and cannot grow to turbulence through either absolute instability or convective instability. They can only stimulate vorticity “rollup” which causes flow transition. The role of all linear unstable modes is to push the vorticity up from the wall (roll up), change the base flow profile to have inflection points. This is particularly important as we found all vortex rings are located in the inflection points. The flow trend to change shear to rotation will occur inside the flow field and the spanwise vortex will form due to the trend from shear to rotation. People have invested many efforts to control the unstable mode growth in order to control flow transition and turbulence. Now, we understand that these efforts are hard to be successful since any factor which can cause vorticity rollup, like gust, dust, noise,

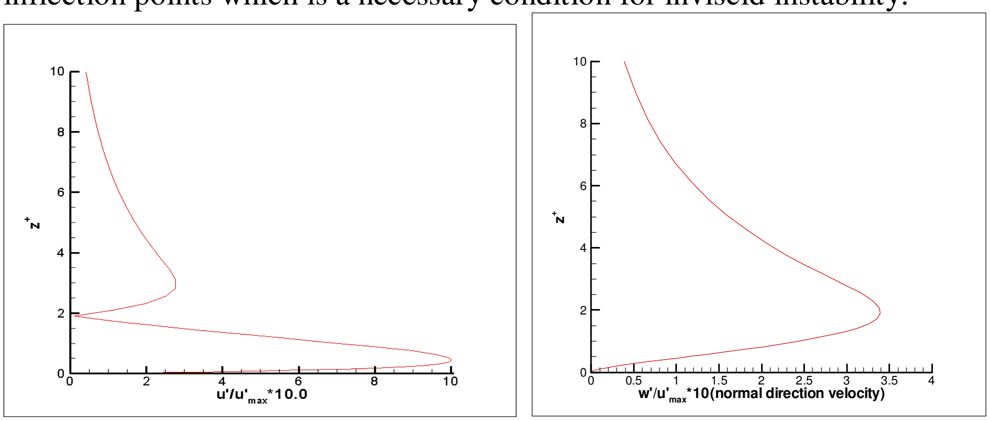
mosquito, fly, can lead to failure of unstable mode control. The key issue is to avoid vorticity "rollup" and shear layer formation.

3) The N-Factor method, which is a good engineering tool to predict flow transition and widely used in aerospace engineering, is based on estimation of the growth rate of linearly unstable modes and thus questionable since there is no direct relation between flow transition and the growth rate of the linearly unstable modes.
4) Since the linear unstable modes cannot form vortex, the non-linear stage is not interaction of 2-D modes and 3-D modes, but spanwise vortex with 3-D modes. The analytic linear solution becomes discrepant from DNS at the beginning. They do not agree with each other even in the very early stages.

## II Preliminary Computational Results and Analysis

### 1. Three inflow disturbances

We have added disturbances at inlet in three ways: 1) enforce 2-D and 3-D Tollmien-Schlichting modes at inflow(Figure 1,) 2) add random noises, 3) add disturbance by small distributed blowing jets, $w' = \varepsilon \sin \omega t \sin \alpha x$ where *w'* is a normalwise disturbance on the wall surface. Preliminary computation shows all three ways can trigger flow transition and the late flow transition structures are very similar to each other. This indicates that all unstable modes play same role that is to trigger vorticity rollup from the wall surface. Further investigation shows these unstable modes are always small (1 or 2 orders smaller than the Blasius base flow.) They cannot form vortex and they cannot cause the flow transition directly. The only role is to push up the vorticity and change the base flow profile to have inflection points which is a necessary condition for inviscid instability.




***(a) u'*** ***(b) w'***

**Figure 1** Tollmien-Schlichting modes (left is streamwise and right is normalwise)

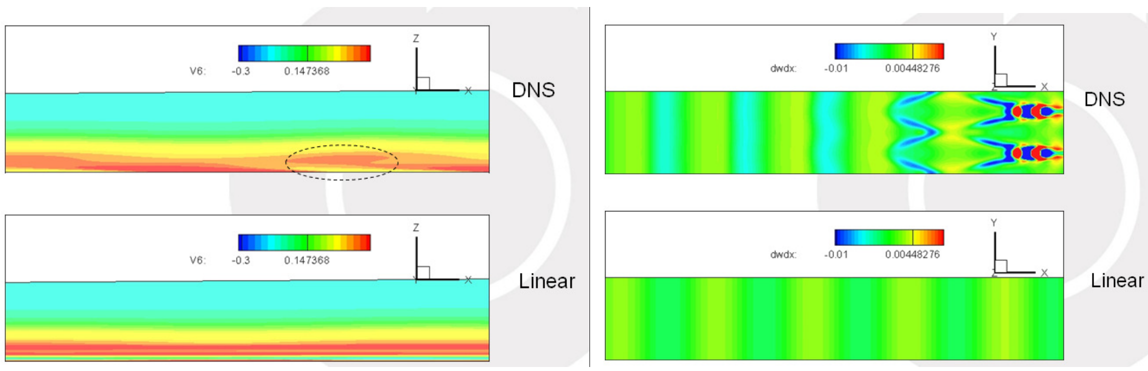
### 2. Comparison of analytic linear solution and DNS solution

We studied flow over a flat plate. The 2-d laminar solution without inflow perturbation is the Blasius solution. After we add some 2-D and 3-D T-S waves at the inflow, the analytic linear solution can be written:

$$u = \bar{u} + u'_{2D} + u'_{3D} = \bar{u} + a_{2D}\varphi_{2D}(z)e^{-i(\omega t - \alpha x)} + a_{3D}\varphi_{3D}(z)e^{-i(\omega t - \alpha x - \beta y)} + CC.$$

where, $\bar{u}$ *is Blasius solution*, $u'_{2D}$ *and* $u'_{3D}$ are 2-D and 3-D perturbation (here T-S waves). We pick the magnitude of perturbation $a_{2D} = 0.03$ *and* $a_{3D} = 0.01$. $\varphi_{2D}(z)$ *and* $\varphi_{3D}(z)$ are 2D and 3D T-S modes. $\omega$ *is real*, $\alpha$ *and* $\beta$ are wave numbers in the streamwise direction x and spanwise direction y respectively. Both $\alpha$ *and* $\beta$ are complex numbers and CC is conjugate part of the perturbation.

Using the analytic linear solution, we can get a time dependent perturbation growth and compare them with our DNS solution. We were expected that when the magnitude of the 2D and 3D T-S waves is small, the analytical linear solution and DNS results are comparable. However, surprisingly, the two solutions quickly depart from each other when the perturbation magnitude is still very small (around 0.03 or even smaller, Figure 2). There is a very short period that we can compare the linear solution with DNS since the role of all linearly unstable modes is not to generate the vortex structure (they cannot), but only generate velocity component in the normal direction, w', which would push the vorticity away from the wall surface. Once the vorticity left the wall surface, the trend from fluid shear to fluid rotation will happen and DNS will differ from the linear solution quickly when Reynolds number is large.




(a) Spanwise vorticty in central plane (b) $\dfrac{\partial w}{\partial x}$ at z=10.95

Figure 2 Comparison of DNS and analytic linear solution

## 3. Important conclusions

It is well known that people spent efforts heavily to remove or reduce the linear unstable modes to control flow transition and further develop laminar wings. However, the role of these modes is only to trigger the wall vorticity rollup and change the base flow velocity profile. These efforts may been very hard to get success since any perturbation could ruin the efforts due to external disturbance by gust, dust, sands, noises, mosquito, fly, etc. In addition, the N-Factor which is based on the growth rate of linear unstable modes is lack of scientific foundation

## 4. Future work

More detailed description of the DNS results and analysis will be given in the final AIAA paper.

## Reference


Liu, C. and Yan, Y., DNS Study of Turbulence Structure in a Boundary layer, AIAA2014-1449, January 13-17, 2014, Maryland, USA