MATH 5307: Mathematical Analysis I

The following list gives topics on which the Preliminary Examination A in Analysis will be based. Math 5307 covers many, but not necessarily all, of these topics. Students (even those who have taken Math 5307) are advised to prepare for the examination using many resources, including (but not limited to) the books suggested below.

The number of lectures per section is only a recommendation and is based on 80-minute lecture time. The exact amount of time spent on each topic will be decided by the instructor.

The Real Number System *(1 lecture)*

- rationals, irrationals, and their properties
- upper and lower bounds, supremum and infimum, maximum and minimum
- completeness axiom
- triangle inequality and Cauchy-Schwarz inequality

Basic Notions of Set Theory *(2 lectures)*

- Cartesian product of sets, ordered pairs
- relations and functions, sequences, composition
- injectivity, surjectivity, bijectivity, inverses
- similar sets, (in)finite sets, (un)countable sets, examples
- set algebra

Elements of Topology *(4 lectures)*

- metric spaces
- open and closed sets, compactness
- adherent points, accumulation points, interior points, exterior points, boundary
- Bolzano-Weierstrass theorem
- Cantor intersection theorem
- Lindelöf covering theorem
- Heine-Borel covering theorem

Limits and Continuity *(6 lectures)*

- convergent sequences in a metric space, Cauchy sequences, completeness
- limits of functions, continuity, uniform continuity
- intermediate-value theorem for continuous functions, discontinuities of real-valued functions
- continuity and inverse images of open or closed sets
- homeomorphisms
- connectedness and arcwise connectedness, components of a metric space
- fixed-point theorem for contractions

Derivatives ***(3 lectures)***

- analytic and geometric notions of derivative, properties of derivative
- one-sided derivatives and infinite derivatives
- local extrema
- mean-value theorem for derivatives, intermediate-value theorem for derivatives
- Taylor's formula with remainder

The Riemann Integral ***(3 lectures)***

- definition and linear properties
- integration by parts and change of variable
- comparison theorems
- integration of continuous functions and functions with discontinuities
- The Fundamental Theorem of Calculus
- Lebesgue's Criterion for Riemann integrability

Infinite Series ***(5 lectures)***

- convergent and divergent sequences of complex numbers
- lim sup and lim inf of a sequence of real numbers
- monotonic sequences of real numbers
- infinite series, inserting and removing parentheses, alternating series
- absolute and conditional convergence
- tests for convergence of series with positive terms
- geometric series
- big oh and little oh notation
- the integral test, ratio test, and root test
- Dirichlet's test, Abel's test
- rearrangements of series
- Riemann's theorem on conditionally convergent series

Sequences and Series of Functions ***(5 lectures)***

- pointwise convergence of sequences of functions; examples
- uniform convergence of sequences of functions; examples
- uniform convergence and continuity
- the Cauchy condition for uniform convergence
- pointwise and uniform convergence of infinite series of functions
- uniform convergence and integration
- uniform convergence and differentiation
- sufficient conditions for uniform convergence of a series.

Some helpful books include

[1] Tom M. Apostol, "*Mathematical Analysis*" (second edition), Addison-Wesley, 1974

[2] Richard A. Goldberg, "*Methods of Real Analysis*" (second edition), Wiley, 1976

[3] Walter Rudin, "*Principles of Mathematical Analysis*" (third edition), McGraw-Hill, 1976