

MATH 5333: Linear Algebra

The following list gives topics on which the Preliminary Examination A in Linear Algebra will be based. Math 5333 covers many, but not necessarily all, of these topics. Students (even those who have taken Math 5333) are advised to prepare for the examination using many resources, including (but not limited to) the books suggested below.

1. Vector Spaces

- (a) definition and properties
- (b) subspaces
- (c) bases, linear (in)dependence, dimension
- (d) sums and direct sums

2. Linear Maps (Transformations)

- (a) definition and properties
- (b) null space (kernel), range, nullity, rank
- (c) matrix of a linear map
- (d) invertibility & application to solving systems of linear equations
- (e) vector space of linear maps

3. Eigenvalues and Eigenvectors

- (a) invariant subspaces
- (b) upper-triangular matrix of a linear map
- (c) diagonal matrix of a linear map
- (d) change of basis

4. Inner-product Spaces

- (a) inner products, norms
- (b) orthonormal bases
- (c) orthogonal projection & minimization problems
- (d) linear functionals and adjoints
- (e) dual vector spaces, bilinear and quadratic forms

5. Operators on Inner-Product Spaces

- (a) self-adjoint and normal operators
- (b) spectral theorems
- (c) isometries

6. Operators on Complex Vector Spaces

- (a) generalized eigenspaces
- (b) the characteristic and minimal polynomials
- (c) Jordan canonical (normal) form

7. Determinant and trace

- (a) definition and properties
- (b) determinant and trace under change of basis
- (c) relationship to the characteristic polynomial
- (d) determinant and volume
- (e) relationship to eigenvalues

Suggested Reading:

S. Axler, Linear Algebra Done Right, 3rdEd., Springer Publishing Co.

C. Curtis, Linear Algebra: An Introductory Approach (Undergraduate Texts in Mathematics), 4th Ed., Springer Verlag, July 1984

S. Lipschutz & M. L. Lipson, Schaum's Outline of Linear Algebra, McGraw Hill Text, 3rd Ed., December 2000

K. M. Hoffmann & R. Kunze, Linear Algebra, 2nd Ed, Prentice Hall Inc., 1971

P. R. Halmos, Finite-dimensional Vector Spaces, 2nd Ed, Springer-Verlag, December 1986