

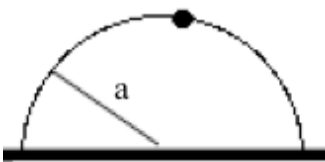
Short Problems:

1. A mass M slides on a horizontal table with frictional coefficient μ is given an initial velocity $V=V_0$ at a starting point ($x=0$). It reaches a frictionless ramp with an incline of 45° and it slides up a distance L along the ramp before stopping and beginning to slide back down. What was the distance the object travelled on the table from its starting point before reaching the foot of the ramp?
2. A rifle of mass 5 kg fires a bullet of mass 0.01 kg with a muzzle velocity of 1000 m/s . The barrel length is 1 m and it is held horizontal to the ground.
 - What is the kinetic energy of the bullet as it leaves the rifle?
 - What was the average force on the bullet while it was in the barrel?
 - If the recoil of the rifle against my shoulder takes $1/2$ a second, what was the average horizontal force that my shoulder exerted on the rifle?
 - The bullet slams horizontally into a wooden block of mass 0.99 kg , sitting on a flat, frictionless surface. What is the speed of the wooden block with the bullet embedded in it?
 - If the bullet is fired straight up, how high does it go (let $g=10\text{ m/s}^2$)?

Long Problems

1. A small bead of mass m slides along a vertical hemispherical loop of radius a as seen in Fig. 1. Assume there is gravity and that \mathbf{g} points straight down perpendicular to the surface.
 - What is the Lagrangian of the system in cartesian coordinates?
 - What are the equations of motion using Lagrange undetermined multipliers?
 - Assume that the Lagrange undetermined multiplier, λ , is a known quantity. What is the force of constraint on the bead in the horizontal direction?

Fig 1



2. A particle moves subject to the potential $U = U_0 \left(r^2 / a^2 \right)$ where a, U_0 are constants and r is the radial distance from the origin in *spherical coordinates*.
 - What is the Lagrangian for this system?
 - Discuss whether this system can be reduced to a two-dimensional system and explain your reasoning.
 - Are there any conserved physical quantities (quantities that do not vary in time)?
 - Draw the effective potential and describe the possible orbits a particle can make in this potential (bound, unbound, types of orbits).