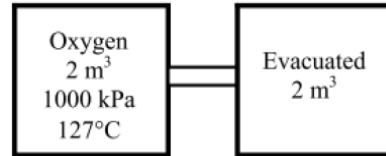


Short Problem 1:

Two adiabatic chambers are connected by a valve. One chamber contains oxygen gas while the other one is evacuated, as shown in the figure. The valve is now opened until the oxygen fills both chambers and both have the same pressure. Determine:

- The change of the total internal energy;
- The pressure in the final chambers;
- The change of the entropy for the system.



(note: the gas constant $R=8.314$ J/mol.)

Short Problem 2

N non-interactive classical particles with a magnetic moment μ are in a uniform magnetic field, B . The particles are fixed on each site of a lattice. The only freedom of a particle is its magnetic moment to be parallel or anti-parallel to the magnetic field ($\mu_i = -\mu$ or μ , $i=1,2,\dots,N$). The system is in equilibrium with the environment at temperature T .

- a) Obtain the Helmholtz free energy;
- b) Obtain the averaged total magnetic moment along the magnetic field,
 $M = \langle \sum_{i=1}^N \mu_i \rangle$, of the system.

Long question

(a) Show that the density of states for a 2-dimensional gas in a 2-dimensional infinite well is given by

$$g(\epsilon)d\epsilon = \frac{mA}{2\pi\hbar^2}d\epsilon \quad \text{where } A \text{ is the area of the 2D well.}$$

HINT - Recall that the energy levels for an infinite well in 2-dimensions is given by

$$\epsilon = \frac{\pi^2 \hbar^2}{2m} \left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} \right) \quad \text{where } n_x \text{ and } n_y \text{ have positive integer values, representing quantization along the } x \text{ and } y\text{-directions respectively, and } L_x \text{ and } L_y \text{ are the lengths of the well.}$$

For simplicity assume a “square well” in which $L_x = L_y = L$ and that $L_x \times L_y = A$, the area of the well.

(b) Using the 2-dimensional density of states given above, calculate the partition function Z of a 2-dimensional gas.

(c) With Z , find the equation of state.

HINT: Recall the Helmholtz free energy F is related to Z through

$$F = -Nk_B T (\ln Z - \ln N + 1) \quad \text{where } N \text{ is the total number of particles.}$$

(d) Find an expression for the internal energy U of the gas using Z . Recall that

$$U = Nk_B T^2 \left(\frac{\partial \ln Z}{\partial T} \right)_A$$

Does this result agree with equipartition theory? Please explain.

Long Problem 2:

The entropy of water at atmospheric pressure and 100°C is 0.31 cal/g·deg, and the entropy of steam at the same temperature and pressure is 1.76 cal/g·deg.

- (i) What is the heat of vaporization at this temperature?
- (ii) The enthalpy ($H = U + PV$) of the steam under these conditions is 640 cal/g. Calculate the enthalpy of water under these conditions.
- (iii) Calculate the Gibbs free energy ($G = H - TS$) of water and steam under this conditions.
- (iv) Prove that Gibbs free energy does not change in a reversible isothermal isobaric (constant P) process.